Real Numbers

Free Response Questions

Q: 1 **Nidaa performed the division of $\frac{53}{83}$ on a calculator whose result is shown below. Even though the quotient has more digits, the calculator didn't display it all due to the limited display area.** **[1]**

Is the output of the division rational or irrational? Explain your answer.

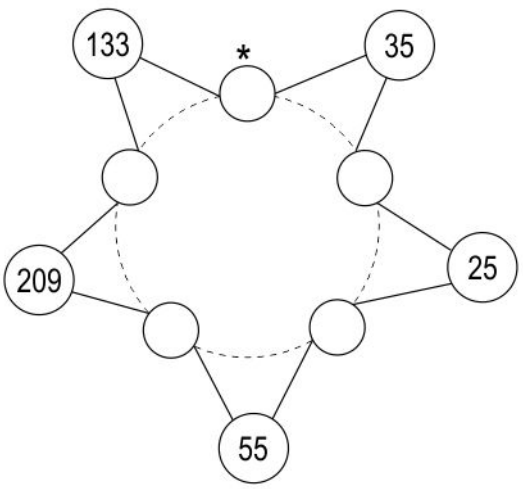
Q: 2 **2^n is not divisible by 6 where n is a positive integer.** **[1]**

Is the above statement true or false? If true, give a valid reason and if false, give an example.

Q: 3 **The prime factorisation of a natural number k is $(3 \times 5 \times p)$ where $p \neq 2$.** **[1]**

What is the prime factorisation of $10k^2$?

Q: 4 **In the figure below, the inner circles are filled with the prime factors of the numbers given in the outer circles. Each number from 1-26 corresponds to the letter in its position in the alphabet, A-Z. For instance, 1 is A, 2 is B, and so on.** **[3]**



Starting clockwise from *, find the word formed by the numbers in the inner circle. Show your work.

Q: 5 **For a positive integer n, m is a prime factor of n.** **[3]**

Show that m is not a factor of (n + 1).

Q: 6 **A rectangular arrangement of pens has rows and columns. Rohan takes away 3 rows of pens and then Sarah takes away 2 columns of pens from the remaining pens. The remaining pens are rearranged in p rows and q columns where p is a prime number.** **[3]**

If Rohan takes 24 pens and Sarah takes 18 pens, find all possible value(s) of p. Show your work.

Case Study

Answer the following questions based on the information given.

Sakshara International School organised a combined exhibition for grade 10 students of its three branches. 345 students from Mumbai branch, 405 students from Pune branch and 270 students from Nagpur branch participated in it. The following were planned for the exhibition:

1. Group projects: The students of each of the three branches were divided in groups for making various group projects such that each group had equal number of students and the number of groups was minimum.

2. Individual project: Each of total 1020 students had to submit an individual project. A fixed number of topics were allotted such that each topic had equal number of students.

3. Inter-state model making competition: A few equal number of students were selected from each branch to participate in the competition. Each branch was supposed to submit between 3 to 7 models.

Q: 7 **Use Euclid's Division Algorithm to find the number of different groups for the group projects. Show your work.** **[2]**

Q: 8 **A maths teacher asked his students to solve the below puzzle regarding the individual project.** **[2]**

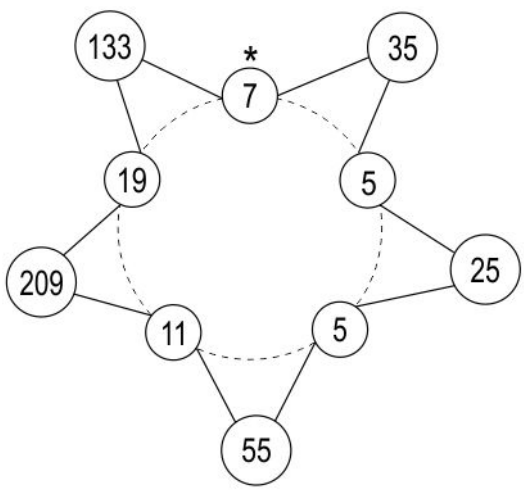
The number of students who got the same topic can be represented as $(2^n \times 5)$ where *n* is a positive integer having the maximum possible value.

Find *n* and the number of topics allotted. Show your work.

Q: 9 **Mumbai branch divided the students selected for inter-state model making competition into the groups of 12 students, Pune into the groups of 10 students and Nagpur into the groups of 15 students.** **[2]**

i) How many students were selected from each branch?
ii) How many models were submitted by individual branches and all the branches together?

Show your work.

Q.No	What to look for	Marks
1	**Writes that output is a rational number.**	**0.5**
	Explains the answer. For example, since the output $\frac{53}{83}$ can be represented in the form $\frac{p}{q}$, where $q \neq 0$, it is a rational number.	**0.5**
2	**Writes that the given statement is true and gives a reason. For example, writes that $2^n = 2 \times 2 \times 2 \times 2 \times \ldots$ n times and hence its only prime factor is 2 whereas prime factors of 6 are 2 and 3.**	**1**
3	**Writes the prime factorisation of 10 k^2 as $(2 \times 3^2 \times 5^3 \times p^2)$.**	**1**
4	**Redraws the factor tree diagram with the prime factors. The tree may look as follows:** 	**2**
	Uses the number to letter mapping and finds the word as GEEKS.	**1**
5	**Uses Euclid's division lemma and writes the equation for some positive integer p as:** $n = mp$ $\Rightarrow p = \frac{n}{m}$	**0.5**

Q.No	What to look for	Marks
	Assumes that *m* is a factor of (*n* + 1), uses Euclid's division lemma and writes the equation for some positive integer *q* as: $n + 1 = mq$	**0.5**
	Rearranges the above equation as: $q = \frac{n}{m} + \frac{1}{m}$	**0.5**
	Writes that *q* is a positive integer, $\frac{n}{m}$ is a positive integer but $\frac{1}{m}$ is not an integer and hence the above equation cannot be true.	**1**
	Hence, uses contradiction to conclude that *m* is not a factor of (*n* + 1).	**0.5**
6	**Writes that in the original rectangular arrangement, there are 8 pens in each row and 12 pens in each column.**	**1**
	Finds the remaining number of pens as 9 rows and 6 columns or 54 pens.	**0.5**
	Writes the prime factorisation of 54 as 2×3^3.	**1**
	Writes all the possible values of *p* as 2 or 3.	**0.5**
7	**Finds HCF of 345, 405 and 270 using Euclid's Division Algorithm as follows:** $405 = 345 \times 1 + 60$ $345 = 60 \times 5 + 45$ $60 = 45 \times 1 + 15$ $45 = 15 \times 3 + 0$ **Finds HCF of 405 and 345 as 15.**	**1**
	Finds HCF of 270 and 15 as follows: $270 = 15 \times 18 + 0$ **Concludes that HCF of 345, 405 and 240 is 15, hence there were 15 students in each group.**	**0.5**

Q.No	What to look for	Marks
	Finds the number of different groups for the group projects as: $\frac{345}{15} + \frac{405}{15} + \frac{270}{15} = 23 + 27 + 18 = 68$	**0.5**
8	**Factorises 1020 as: $2^2 \times 3 \times 5 \times 17$.** **From the above factorisation concludes that $2^n \times 5 = 2^2 \times 5$ and hence finds the maximum possible value of *n* as 2.**	**1**
	Finds the number of topics allotted as the remaining factors of 1020 as: **$3 \times 17 = 51$**	**1**
9	**i) Finds LCM of 12, 10 and 15 using prime factorisation as:** **$12 = 2 \times 2 \times 3$** **$10 = 2 \times 5$** **$15 = 3 \times 5$** **LCM $= 2^2 \times 3 \times 5 = 60$** **Concludes that 60 students were selected from each branch for the inter-state model making competition.**	**1**
	ii) Finds the number of models submitted by each branch as: **Mumbai $= \frac{60}{12} = 5$** **Pune $= \frac{60}{10} = 6$** **Nagpur $= \frac{60}{15} = 4$** **Finds the total number of models submitted as 15.**	**1**